6.1b Euler's Method

used to find numerical solutions to d.e.'s

d.e. $\frac{dy}{dx} =$ ——— solve the de: find y

i.c. (x_0, y_0) ← initial value

algebraic: $y = f(x)$

numerical: find $f(x_n)$ ← final value (x_n, y_n)

i.c. $y_0 = f(x_0)$

graphical:

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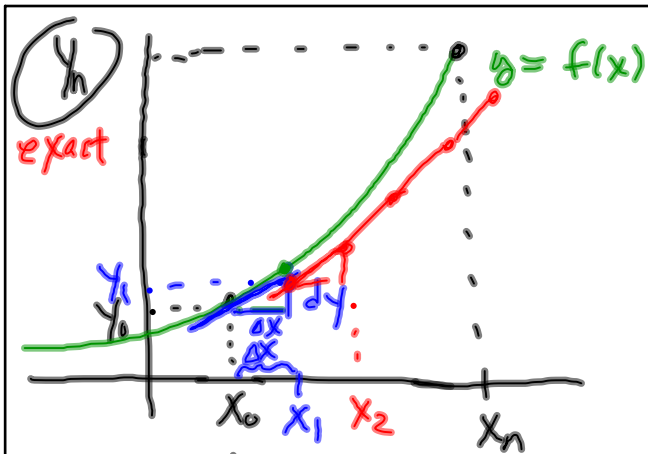

given (x_0, y_0), $\frac{dy}{dx}$

line ' curve '

$dy \approx \Delta y$

$dy = \frac{dy}{dx} \cdot dx = \frac{dy}{dx} \Delta x$

new y = old y + $\frac{dy}{dx} \cdot \Delta x$

$$y_{n+1} = y_n + f'(x,y)\,\Delta x$$

Euler's formula

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Ex 1

$\frac{dy}{dx} = x + y$ $f(2) = 0$ $f(3) = ?$

$x = 2$ $x = 3$ given

$y = 0$ $y = ?$ 3.4695 $\Delta x = .2$

new old approx.

$y_1 = 0 + 2 \cdot (.2) = .4$

$y_2 = .4 + 2.6\,(.2) = .92$

$y_3 =$

x	y	y'
2	0	2
2.2	.4	2.6
2.4	.92	3.32
2.6	1.584	4.184
2.8	2.9208	5.2248
3	3.4695	

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Ex 2

$\frac{dy}{dx} = 2x - y$ $x = 2$ $y = 3$ $x = 1.5$ $y = ?$

$\Delta x = -.1$

$y_1 = 3 + 1 \cdot (-.1) = 2.9$

$y_2 = 2.9 + .9\,(-.1) =$

x	y	y'
2	3	1
1.9	2.9	.9
1.8	2.81	
1.7		
1.6		
1.5	2.61051	

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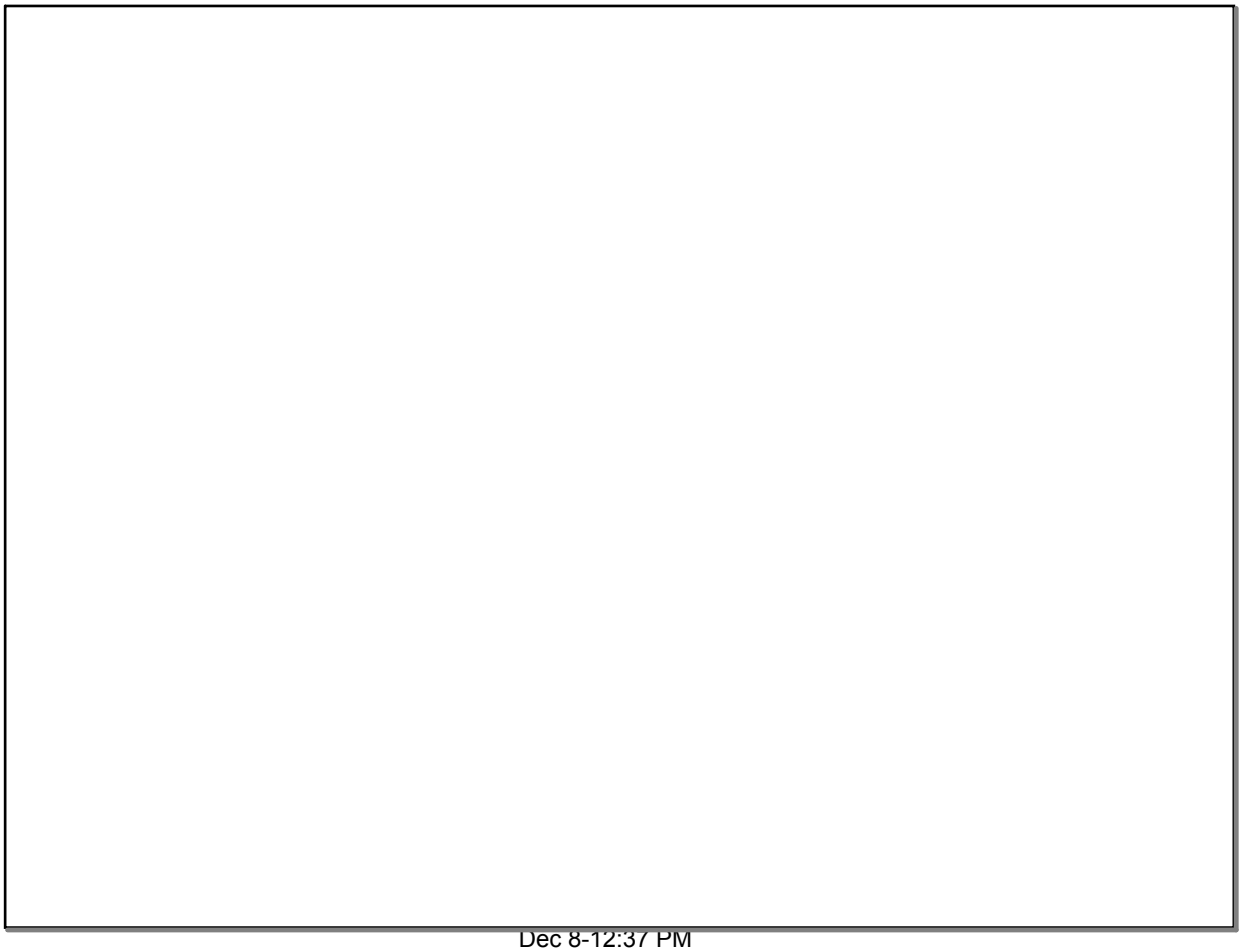

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